

## 6a Calculation of number of “messages” sent by market

Assume a company produces two products, product 1 in quantities  $d_1$  per WIP inventory turn, and product 2 in quantities  $d_2$  per WIP inventory turn, where  $d_1 + d_2 = D$  total units produced per WIP inventory turn. The actual demand of the market for the two products is random, and results in a variety of possible sequences such as:

1121221122212212  
 2211212211121221  
 2122122111211212  
 , etc.

The market makes  $D$  Choices per WIP inventory turn of either 1 or 2. Each sequence is a “state” of the market or complexion in the sense of Gibbs<sup>i</sup>. The number of *distinct* states or “messages” sent by the market, to be satisfied by the company, is calculated by the usual combinatorial formula<sup>ii</sup>:

$$\text{Number of Distinct Messages per inventory turn} = M = \frac{D!}{d_1!d_2!} = \binom{D}{d_1} = \frac{D!}{d_1!(D-d_1)!} \quad (0.1)$$

We will follow Boltzmann by taking the logarithm of the number of states, which in the Microeconomic case is the number of distinct messages from the market: According to Stirling’s formula, to first order<sup>iii</sup>:

$$\log_2 D! \cong D \log_2 D - D, \text{ note that } D = (D-d_1) + d_1 = d_2 + d_1$$

$$\log_2 M = (D \log_2 D - d_1 \log_2 d_1 - (D-d_1) \log_2 (D-d_1))$$

$$\log_2 M = ((D-d_1) + d_1) \log_2 D - d_1 \log_2 d_1 - (D-d_1) \log_2 (D-d_1)$$

$$\log_2 M = - \left( (D-d_1) \log_2 \left( \frac{D-d_1}{D} \right) + d_1 \log_2 \left( \frac{d_1}{D} \right) \right), \text{ multiplying by } \frac{D}{D}, \text{ obtain:}$$

$$\log_2 M = -D \left( \left( \frac{D-d_1}{D} \right) \log_2 \left( \frac{D-d_1}{D} \right) + \left( \frac{d_1}{D} \right) \log_2 \left( \frac{d_1}{D} \right) \right), \text{ let } p_1 = \frac{d_1}{D}, p_2 = \frac{D-d_1}{D}$$

$$\log_2 M = D \left( - \{ p_1 \log_2 p_1 + p_2 \log_2 p_2 \} \right) \rightarrow D \left\{ - \sum_{i=1}^m p_i \log_2 p_i \right\} = DH_m \text{ for } m \text{ products,}$$

$$M = 2^{DH_m} = \text{Number of Distinct Messages } M \text{ per WIP inventory turn} \quad (0.2)$$

$$DH_m = \log_2 M$$

$$D = \frac{\log_2 M}{H_m} \quad (7.3a)$$

Notice that Shannon's equation for Information emerged naturally. The market is making  $D$  variety choices per WIP inventory turn, selected from one of the  $m$  products, each of the  $D$  events per turn containing information of  $H_m$  bits. The  $M$  messages per month corresponds to the number of unique states per month.

$$H_m = - \sum_{i=1}^m p_i \log p_i = \text{Shannon Information in Bits per Choice}$$

$$\text{Transmission Rate of Market} \rightarrow D H_m \rightarrow \left( \frac{\text{Choices}}{\text{WIP Turn}} \right) \left( \frac{\text{Bits}}{\text{Choice}} \right) \rightarrow \text{Bits per WIP Turn} \quad (0.3)$$

Thus the market is acting like a communication system, transmitting  $D H_m$  bits of information per WIP turn about the variety of products it wants to buy which the company presently offers. Referring to the early automotive market, initially the market demanded utility transportation and Ford responded with  $m=1$  in the form of the Model T. As the technology of cars improved from 1908 to 1925, Ford continued on with  $m=1$  whereas the market demanded variety as brilliantly offered by Sloan of GM where  $m>5$ , and the seemingly impregnable Model T was quickly destroyed<sup>iv</sup>. Thus the market began sending more complex messages, which will be discussed in the next section.

*Toyota Production System:* We can now apply the goal of the Toyota Production System in to a market driven criterion: The goal of the Toyota Production System is to reduce  $W$  such that the amount of information in a factory is equal to that needed to produce any part number demanded by the market, *and no more*. The production of  $m$  different end items needed to compete with GM etc requires  $Q$  different items of WIP. Each of the  $D$  choices the market makes per WIP turn carries  $H_m$  bits of information in (0.3) which translates through the explosion of the Bills of Material to  $H_Q$  bits of internal information.

<sup>i</sup> Gibbs, J Willard (1981 reprint) *Elementary Principles in Statistical Mechanics*, Ox Bow Press

<sup>ii</sup> Walpole, Ronald et al (2002) *Probability and Statistics for Engineers and Scientists* p.37

<sup>iii</sup> Stirling's formula  $\log D! = D \log D - D$  is only in error by 1% when the number of products shipped per month is  $D=10$ , and of course is entirely negligible for most companies when  $D \gg 10$ . See Reif, F 1965, *Fundamentals of Statistical and Thermal Physics*, pp 613-614 for an investigation of the accuracy of Stirling's formula.

<sup>iv</sup> MIT LAI paper 2007 op cit