

6c Lemma 1:log of sum of two numbers

When we examine the total Work In Process W of a factory or transactional process, we find that it consists of Q different types of items or sub-products in process, or different tasks not yet completed. Then we can write:

$$W = w_1 + w_2 + \dots + w_Q = \sum_{i=1}^Q w_i \text{ where } w_i \text{ is the number of units}$$

of the i^{th} subproduct or task type in WIP

We will first derive an expression for $Q = 2$ and then generalize:

$$W = w_1 + w_2$$

Thus far we have used the natural logarithm $\log W = \log_e W$. We will take the $\log_2 W = 1.44 \log_e W$ and use the conversion factor where needed.

This allows results to be stated in bits rather than nats. We can write:

$$\log_2 W = \frac{w_1 + w_2}{W} \log_2 W = \frac{w_1}{W} \log_2 W + \frac{w_2}{W} \log_2 W = -\frac{w_1}{W} \log_2 \left(\frac{1}{W} \right) - \frac{w_2}{W} \log_2 \left(\frac{1}{W} \right),$$

we will now add $0 + 0$

$$\log_2 W = -\frac{w_1}{W} \log_2 \left(\frac{1}{W} \right) - \frac{w_2}{W} \log_2 \left(\frac{1}{W} \right) + \left(\frac{w_1}{W} \log_2 w_1 - \frac{w_1}{W} \log_2 w_1 \right) + \left(\frac{w_2}{W} \log_2 w_2 - \frac{w_2}{W} \log_2 w_2 \right)$$

$$\log_2 W = -\frac{w_1}{W} \log_2 \left(\frac{w_1}{W} \right) - \frac{w_2}{W} \log_2 \left(\frac{w_2}{W} \right) + \frac{w_1}{W} \log_2 w_1 + \frac{w_2}{W} \log_2 w_2$$

which can be generalized from $Q = 2$ to Q

different types which comprise W by defining the Probability that a unit of WIP

is the i^{th} product as $p_i = \frac{w_i}{W}$

$$\log_2 W = -\sum_{i=1}^Q p_i \log_2 p_i + \sum_{i=1}^Q p_i \log_2 w_i = H_Q + \sum_{i=1}^Q p_i \log_2 w_i \quad (0.1)$$

$\log_2 W = H_Q + \epsilon \log_2 w_i$, where ϵ is the expectation:

$$\epsilon \log_2 w_i = \sum_{i=1}^Q p_i \log_2 w_i$$

$$W = 2^{H_Q + \epsilon \log_2 w_i} \quad (0.2)$$

Since lead time is proportional W , it is also proportional to the exponent of process entropy. The portion of the entropy related to H_Q is the response of the company to the variety of product or service demanded by the marketplace. The balance of the entropy $\epsilon \log_2 w_i$ are due to inefficiencies in the company's process in response to this demand for variety. Both forms of entropy can be reduced by process improvement. While the entropy within a process carries no economic value, entropy creates economic waste when it emerges as product

