Log Cycle Time as a Predictor of Cost Reduction

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Abstract

From the time of Henry Ford it has been known that large reductions in cost result from radical reductions in process cycle time. In the case of a Model T, a reduction in cycle time from 14 days to 33hours (i.e. >90% reduction) allowed the same car to be sold at $345 vs. $850. It would be of great benefit if management could predict the cost reduction and resulting profit that would flow from investments in process improvement initiatives such as Lean, Six Sigma and Complexity reduction which reduce cycle time and improve quality. Empirical data indicates that cost reduction due to waste elimination is consistent with the log of the ratio of cycle time reduction.

Little's Law governs the average cycle time τ of any process, and is equal to the number of units of Work In Process divided by the Average Completion Rate. Little's Law, when treated as a dynamical equation, results in an expression for process entropy which is also proportional to the log of the ratio of cycle time and WIP reduction at constant volume. Thus the *entropy* in an economic process follows the same log function as *entropy* and *waste* in a Carnot Heat engine. We hypothesize that the *waste* in an economic process is also proportional to entropy as in a Carnot engine.

The logarithm of Work In Process W, consisting of Q different part numbers with w_i units of the i^{th} part number , can be shown to be equal to:

$$\log_2 W = -\sum_{i=1}^{Q} p_i \log_2 p_i + \sum_{i=1}^{Q} p_i \log_2 w_i = H_Q + H_{PD} \text{ where } p_i = \frac{w_i}{W} \text{ and } W = \sum_{i=1}^{Q} w_i$$

H_Q is the Shannon Entropy due to Variety which is diminished by complexity reduction, and H_{PD} is Entropy due to Process Deficiencies such as defects, setup time, etc which can be reduced by process improvement (e.g. the Toyota Production system). We will test if the hypothesis that the waste is proportional to the logarithm of the ratio of final to initial cycle time, Cost = $k\log(\tau_f/\tau i)+1$, is confirmed by experiment. If so, it would provide management a powerful tool to predict the cost reduction due to investments in process improvement.

A practical procedure for necessary data collection is defined which will allow management to predict cost reduction due to process improvement. Additional case studies will test the validity of this Equation of Cost Reduction in which academics are invited to participate.

Keywords: Equation of Projected Cost Reduction; Process Entropy; Information; Complexity; Waste; Little's Law; Shannon; Boltzmann; Carnot

Classification-JEL: C10,C11,C13,C24,C51,D1-D4, D8, L6, L7, L8, L11, L15, M21

Section 1: Empirical Observations

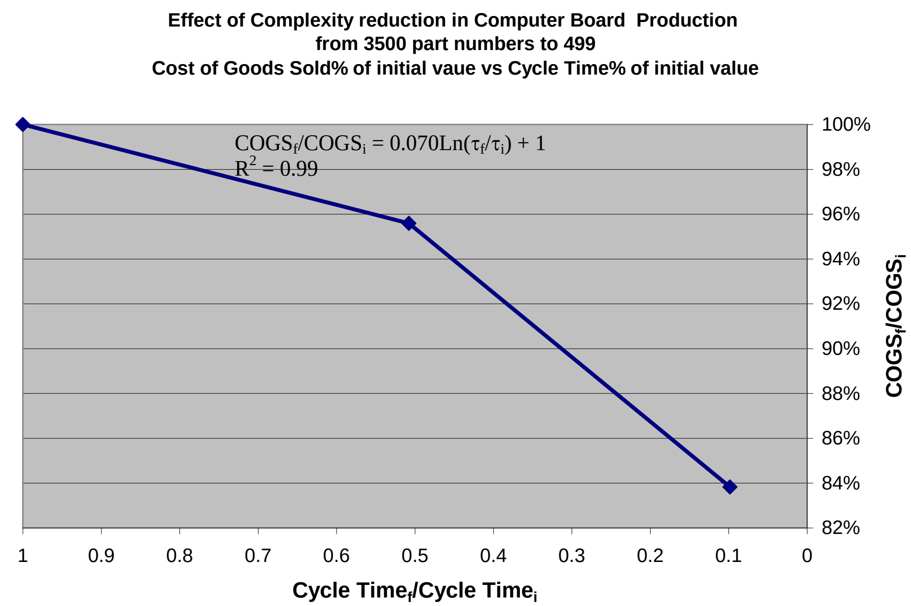
Case Studies show that small reductions in cycle time yield modest cost reductions. As cycle time is reduced to about 30% of its original value, the Cost of Goods Sold dramatically falls. This result was unexpected and contrary to the conventional wisdom of "low hanging fruit". The best fit ($R^2 > 0.94$) to the data is a log curve:

$$\textbf{\% Cost Reduction} \cong \mathbf{k\log(\tau_f/\tau_i)+1} \qquad (1.1)$$

Where τ_i is the initial cycle time and τ_f is the final cycle time after process improvement, where $\tau_i > \tau_f$ hence the first term in (1.1) is always negative. The cost falls as cycle time is reduced due to waste elimination. Waste is defined as any cost that does not add a form, feature or function of value to the customer. The first term in (1.1) is the amount of waste that is eliminated by process improvement defined below...

Case Study 1: (Client name withheld): External Complexity reduction

A $ 2.3 Billion revenue computer products company was losing money on a product line that consisted of $m_{initial}$ =3500 different end items. To reduce the cost of complexity, the new CEO reduced the number of part numbers offered to customers to m_{final} =499. The gross profit increased from 32% to 43%.

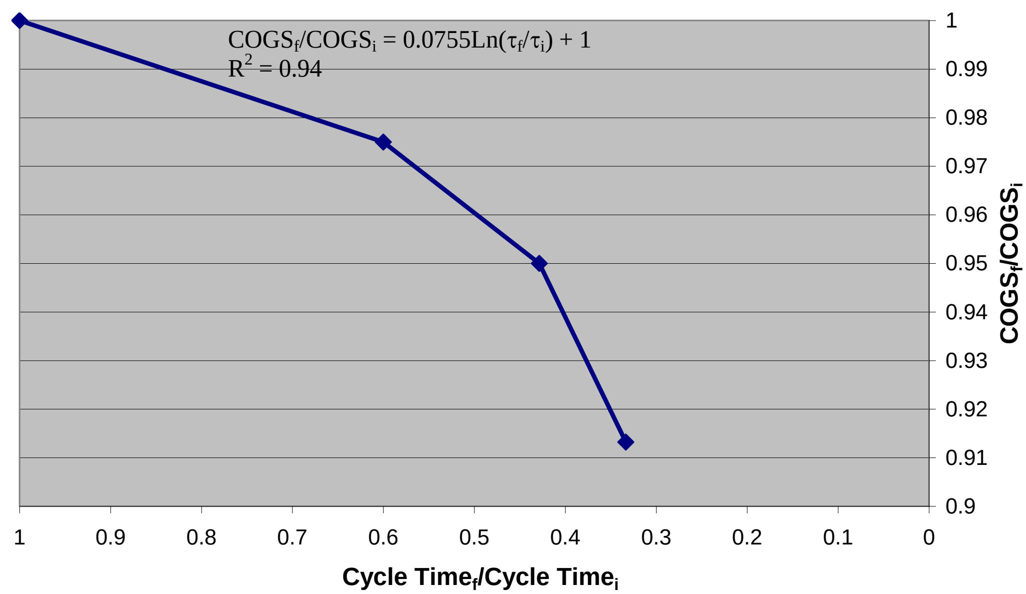


Cost of Goods Sold fell in proportion to waste reduction of $\mathbf{k\log_e(\tau_f/\tau_i)+1}$ with k≅0.07

Case Study 2: United Technologies Automotive, H&F div(PTG): Lean Six Sigma

The company produced m=168 different hose and fittings products with an average cost per part of $50 and operated at 10.5% Gross Profit Margin. Because internal components were qual tested and approved by clients such as Ford, GM etc negligible opportunity for internal complexity reduction existed. Rather, Work In Process was reduced via the Toyota Production system known in America as Lean Six Sigma. Reduction of setup time by 70% resulted in cycle time reduction of 70. The result of cost reduction and revenue growth resulting from cycle time reduction resulted in Earnings Before Interest, Taxes and D&A (EBITDA) growing from $10.4 Million to $46.7 million in three years. The setup time at key workstations was reduced from an average of 2 hours to approximately 10 minutes. The resulting gross margin increased from 12.0% to 19.5%. Operating margin grew from 5.4% to 13.8%. Sales grew from $144 million to $311million per year. Cost of Goods Sold rose from $127.4 Million to $250.6Million.Product complexity was essentially constant.

Effect of reduction of Process Deficiencies in Hose and Fittings production
Cost of Goods Sold% of initial vs Cycle Time% of initial



Note again that the percentage by which Cost of Goods Sold fell in proportion to klog$(\tau_f/\tau_i)+1$ with k≅ 0.076

Discussion: One of the major items of non value add cost that was eliminated was a warehouse comparable in size to the factory. When cycle time fell below 5 days demanded by customers the warehouse could be closed leading to a quantum of cost reduction. If cost reductions are delayed by lease obligations of failure of management initiative, the cost reduction will be delayed and introduce noise.

Critique: US companies only report Work In Process once per year. To expand the data set of observations, several firms will be instrumented to report Work In Process every month in all processes, including Product Development, Manufacturing, Distribution, etc and is discussed at the end of the paper.

Preliminary Conclusion:

$$\textbf{\% Cost Reduction} = k_B \log_e(\tau_f/\tau_i) \cong k_B \log_e(\alpha\tau_f/\tau_i) \qquad (1.2)$$

Where α=(Revenue)$_i$/(Revenue)$_f$ which we will show in Section 5 nulls out the effect of growth in unit volume and its surrogate, revenue growth. The maximum cost reduction possible is determined by the maximum reduction in cycle time which equals the sum of value add times down the critical path. The factor $\mathbf{k_B} \cong 0.07$ was found from manufacturing examples, and it is likely that different factors will be appropriate to transactional processes. The important hypothesis is that waste cost is a log function of the ratio of cycle times. We will find that the $\log_2$ expression is more convenient in subsequent work in which $\mathbf{k_B} \cong 0.1$.

The results thus far are tentative but important enough to try to understand why waste follows a log curve, and to obtain more data than is possible using publicly available financial reports as discussed in the Critique. We will now investigate *why* cost reduction is proportional to the log of the ratio of process cycle times.

Section 2: Cycle Time of a Process: Little's Law

The major intuitive insight of Henry Ford was that the elimination of waste in labor and overhead by process improvement drove shorter cycle time. The Model T originally sold for $850 and took 14 days to produce. Process improvement eliminated nearly all the waste in labor and overhead, and the same car was produced in 33 hours and sold for $345 at far higher total profit[2]. Toyota adapted this process to the production of a variety of cars using what was originally called the Toyota Production System which has been generalized[3] to include non-manufacturing processes and is now known as Lean Six Sigma.

Since the cycle time of a process is found empirically to drive process cost, it is important to study the drivers of process Cycle time. The average Cycle time of any process is governed by Little's Law[4]. The Average Cycle time, per cycle of production, from injection of work into a process to that work's completion is:

$$\text{Avg. Cycle Time of any Process}=\frac{\text{Number of Units of Work In Process}}{\text{Average Completion Rate}}=\frac{W}{D}=\tau \qquad (2.1)$$
$$= \text{time/completion of a cycle of production}$$

As an example of Little's Law, if a process has Work In Process (WIP) of 50 units and has an average completion rate of 2 units per hour, then the average time for a unit of WIP to transit the process from injection to completion is:

$$\text{Avg. Cycle Time of Process} = \frac{50\text{ units}}{2\text{ units/hour}} = 25\text{ hours}$$

The average completion rate D in (2.1) is the customer Demand rate, and hence is exogenous to the process. Notice that the Work In Process in Little's Law is a dimensionless number…it is simply the number of units, not their dollars of cost or revenue etc. Though the WIP may consist of a variety of different items having different processing times but only the *average* completion rate D governs the cycle time of the process. Moreover, Little's Law is distribution independent: whether task processing times follow a Gaussian distribution as in manufacturing, a Rayleigh distribution as in product development, whether arrivals/departures are Poisson is irrelevant to cycle time. Little's Law is quite robust, and has been referred to as the Newton's 2^{nd} Law of processes by Professors Hopp and Spearman[4].

If the company has W units of Work In Process Inventory[5] and ships products which contain C units per year, then the company turns inventory Z=C/W times per year. The time per inventory turn will be the interval of interest in subsequent calculations.
To reduce the waste in a process, empirical evidence shows that we must reduce the cycle time, or equivalently, accelerate the velocity of the process. We can use Little's Law to determine the energy needed to reduce the cycle time of a process and compare the result to (1.1). To compute this energy, we transform (2.1) into an equation of velocity by inversion:

$$\text{Velocity} = \frac{D}{W} = \text{cycles of work in process completed/unit time} \qquad (2.2)$$

To accelerate velocity requires an external force acting between the initial velocity v_i and final velocity v_f. In Online Appendix 2 we show that the energy needed to accelerate a process is proportional to:

$$\text{Process Improvement Energy} = -D^2 \log\left(\tau_f/\tau_i\right) \qquad (2.3)$$

Which in form is consistent with (1.1). For future reference it is also of interest to note that Equation (2.3) is also of the same form as the energy needed to compress an ideal gas and is proportional to the energy waste in a thermodynamic heat engine (Online Appendix 1).

Conclusion:

The empirical observation that process waste falls as a function of **log(τ_f/τ_i)** identified in (1.1) has as its source the energy a function of - **log(τ_f/τ_i)** needed to accelerate cycle time by process improvement and thus the log dependence of waste is less likely to be a mere coincidence.

Section 3: Information = Negative Entropy→ Reduction of Waste

To investigate the nature of this energy of process acceleration, let us compute $\log\tau = \log(W/D) \rightarrow \log W$ since D is an exogenous constant. When we examine the Work In Process W of a factory or transactional process, we find that it consists of Q different types of items or sub-products in process, or different tasks not yet completed. The total Work In Process W is the sum of these Q different items, the i^{th} of which has quantity w_i:

$$W = w_1 + w_2 + \ldots w_Q = \sum_{i=1}^{Q} w_i$$

We therefore need to compute logW and can use any base. We choose base 2 for reasons that will become clear. In Online Appendix 3 we show that:

$$\log_2 W = -\sum_{i=1}^{Q} p_i \log_2 p_i + \sum_{i=1}^{Q} p_i \log_2 w_i = H_Q + H_{PD} \qquad (3.1)$$

$$\text{Therefore } W = 2^{H_Q + H_{PD}} \text{ via Little's Law}$$

$$\tau = \frac{W}{D} = \frac{2^{H_Q + H_{PD}}}{D}$$

In Online Appendix 4 we show that H_Q is known as Shannon Entropy and is measured in bits of Information. Entropy is a measure of the variety of products or offerings. While variety may increase revenue, it will certainly increase cost and waste in a process. We will see that the portion of the entropy related to H_Q is caused by the response of the company to the variety of product or service demanded by the marketplace. The balance of the entropy $H_{PD} = \sum_{i=1}^{Q} p_i \log_2 w_i$ is due to Process Deficiencies in the company's process such as long setup time, defects, etc, which cause waste in response to this demand for variety. Both forms of entropy can be reduced by applying appropriate process improvement tools[3].

Section 4: Information and Process Entropy

The term $-\sum_{i=1}^{Q} p_i \log_2 p_i = H_Q$ in (3.1) is identical to the Boltzmann expression for thermodynamic entropy[6] $S = -k\sum_{i=1}^{Q} p_i \log_2 p_i$ in an engine but with k=1. Thus the nature of the work required for the reduction of logW necessary to accelerate the process and eliminate waste is the increase in information added by process improvement to reduce process entropy. Shannon's equation of information will be developed from first principles in Online Appendix 4. Since H_Q (3.1) is entropy in bits, the term $H_{PD} = \sum_{i=1}^{Q} p_i \log_2 w_i$ must also be in units of bits. Hence we define (3.1) as the *Process Entropy*.

Discussion of Terms in the Process Entropy, equation (3.1):

We can most easily explain the role of each term in (3.1) by considering limiting cases. *Complexity:* Let us assume that each of the Q items of WIP W had about the same quantity of units in WIP, $w_i \cong W/Q$. Then the probability of occurrence of the i^{th} item is $p_i \cong w_i/W = 1/Q$ and:

$$H_Q = -\sum_{i=1}^{Q} p_i \log_2 p_i = -\sum_{i=1}^{Q} \frac{1}{Q}\log_2\left(\frac{1}{Q}\right) = -\left\{\frac{1}{Q}\log_2\left(\frac{1}{Q}\right) + \frac{1}{Q}\log_2\left(\frac{1}{Q}\right)\cdots Q \text{ terms}\right\} = \log_2 Q \quad (4.1)$$

Therefore, *H measures the variety of internal products in WIP needed to deliver m different end products to the customer*. H can be reduced by reducing complexity Q through internal standardization. For example, International Power Machines[7] reduced the number of internal part numbers to produce a fixed breadth of external product line from approximately Q=1000 internal part numbers to 260. For approximately uniform usage, $H_Q \sim \log_2 1000 = 10$, whereas $H_{Intrinsic} \sim \log_2 260 = 8$, where Intrinsic refers to a minimum irreducible set of components. In addition, the reduction of Q reduces WIP as is seen in Section 5 below and hence the entropy due to the second term in (3.1). The gross profit margin increased from 18% to 37% in consequence of the reduction of waste. The larger is Q and H, the more setups will be required to meet demand, hence the greater the non value add waste of setup time, and accompanying scrap as well as the cost of tooling, dies, etc. Recall that value add costs add a form, feature or function valued by the customer. All else is waste, e.g. the cost of setup, scrap, rework, warehousing, distribution, some labor and most overhead cost. The minimum amount of non value add cost is determined by the well known Value Stream Mapping process[3]. As Q is reduced, more volume is driven through fewer part numbers leading to lower procurement costs, with similar impact on non-manufacturing processes.

Lean: The second term in (3.1) can similarly be understood. Assume that $p_i \cong 1/Q$, $w_i \cong W/Q$, then:

$$H_{PD} = \sum_{i=1}^{Q} p_i \log_2 w_i = \sum_{i=1}^{Q} \left(\frac{1}{Q}\right) \log_2 \left(\frac{W}{Q}\right) = \left(\frac{1}{Q}\right) \log_2 \left(\frac{W}{Q}\right) + \cdots Q\text{terms} = \log_2 \left(\frac{W}{Q}\right) \qquad (4.2)$$

Therefore the *second term* in (3.1) $\sum_{i=1}^{Q} p_i \log_2 w_i = \varepsilon \log_2 w_i$ where ε stands for expectation is the average of *the log of WIP per part number*. Thus the larger is H_{PD}, the larger will be the waste due to scrap, rework, obsolescence, warehouses, distribution centers, transport, and IT systems, and all related indirect personnel to control and store all the material as well as expediting expense to compensate for long cycle times. We will see in Section 6 below that in manufacturing this $\varepsilon \log_2 w_i$ term is primarily driven by setup time, machine downtime, and quality defects and other *process deficiencies*. Thus equation (3.1) for Process Entropy can be written as:

$$\text{Process Entropy} = H_Q + H_{PD} = \text{Entropy of Variety} + \text{Entropy of Process Deficiencies} \qquad (4.3)$$

Conclusion:
Entropy of the form $\log\left(\tau_f/\tau_i\right)$ probably plays the same waste creating role in business processes that entropy creates in engines, of the form $\log\left(V_f/V_i\right)$, which is derived in Appendix 1.

Section 5: Process Improvement and the reduction of WIP

To understand how improvement process reduces Work In Process and hence cycle time and entropy, it is instructive to consider the two principal expressions for WIP as a function of process parameters:

Equation of Manufacturing Work In Process: The minimum WIP in a factory that is consistent with a given demand D has been derived by Patell and George[8], and a representative equation is:

$$\text{FactoryWIP} \geq \frac{QAsD}{1-X-\zeta D} + QA \qquad (5.1)$$

Where s=setup time, A=number of workstations in the process, X=Defect rate, ζ=Processing time per unit. One can see that reducing the number of different internal part numbers Q by 50% reduces WIP by 50%. Note that WIP is proportional to demand D. If we wish to compare the relative WIP in two different periods, we must null out the effect of revenue growth by multiplying final WIP by the α factor in (1.2). The factor ζD is the percent of capacity utilized and may be assumed constant for the present analysis. Thus factory WIP is proportional to D and can be nulled out as described.. The reduction of work in process is possible due to the application of specific tools[3] such as the implementation of Pull systems to synch up WIP with demand, the Four Step Rapid Setup method to reduce s, and a battery of quality tools to reduce X, together with complexity reduction tools to reduce Q. Traditional manufacturing engineering focused on reducing ζ, often through time and motion studies[9], automation, etc. The application of each tool is effectively the addition of Information to the process which reduces WIP and hence process entropy in (3.1).
If the setup time s can be driven toward zero, as is the goal of Toyota, then according to the Patell-George equation below, w_i=1, and since log(1)=0, (3.1) becomes:

$$W=2^{H_Q+\varepsilon\log_2 w_i}=2^{H_Q}2^{\varepsilon\log_2 w_i}=2^{H_Q}2^0=2^{H_Q} \qquad (5.2)$$

In such an instance, there is only one unit per part number hence $p_i \equiv 1/Q$, $H \equiv \log_2 Q$ and $W \rightarrow QA$ in (5.1) as required. In this instance logQ→ Process Entropy.

In the case of Henry Ford's Model T, there was no variety of product offered to customers, and each workstation produced only one part number hence setup time s≡0 in (5.1) and logQ=H. Thus it can be seen that Toyota approaches the low entropy and waste of Ford while providing variety to customers. Thus adding information to the process to reduce setup time, defects, etc reduces Process entropy and waste.

Equation of Transactional Work In Process: Non manufacturing processes such as Product Development, Marketing, Planning, and most Government and Defense processes, are dominated by non-repetitive tasks. The Work In Process is approximated by the Extended Pollaczek-Khintchine equation[3]:

$$\text{WIP = No.of Tasks In Process} \cong \left(\frac{1}{K+1}\right)\left(\frac{\rho^2\{1+X\}^2}{1-\rho\{1+X\}}\right)\left(\frac{C_S^2+C_A^2}{2}\right) \qquad (5.3)$$

ρ = % of maximum capacity utilized

K= Number of resources cross trained

X= % defectives that must be reworked

$$C_A = \frac{\text{Standard Deviation of time between arrival of tasks}}{\text{Mean Time between arrival of tasks}}$$

$$C_S = \frac{\text{Standard Deviation of time to perform tasks}}{\text{Mean Time to perform tasks}}$$

Specific tools to reduce WIP include modularization of designs to reduce ρ and Cs, cross training to increase K, Open Innovation to reduce ρ and C_A, etc (see *Fast Innovation*[3]). Each tool adds information to the process thus reducing WIP, entropy, and waste.

Section 6: Information Transmission by the Market Revenue Stream

The company responds to demand from the market by offering complexity of m different products. In Online Appendix 4 we show that, per inventory turn, the market is transmitting DH_m bits, where $H_M = -\sum_{i=1}^{Q} p_i \log_2 p_i$ and pi is the probability that the market will demand the i^{th} product.. Thus the market is acting like a communication system, transmitting DH_M bits of information per WIP turn about the variety of products it wants to buy which the company presently offers. Referring to the early automotive market, initially the market demanded utility transportation and Ford responded with m =1 in the form of the Model T. As the technology of cars improved from 1908 to 1925, Ford continued on with m=1 whereas the market demanded variety as brilliantly offered by Sloan of GM where m>5, and the seemingly impregnable Model T was quickly destroyed[10]. Thus the market began sending more complex messages, which Ford refused to receive due to his myopic view that an efficient process had to be dedicated to a single product. Toyota showed that Ford's low cost could be approached while offering a variety of products.

Toyota Production System: The breakthrough that Toyota achieved can be best illustrated by a single process improvement tool, the Four Step Rapid Setup method, e.g. the setup time of a 2000 ton press is reduced from 4 hours to less than 10 minutes[2]. Applied to (5.1) this would reduce WIP by a factor of 24…not as low in cost as Henry Ford but allowing the efficient production of a variety of autos. The goal of the Toyota Production System is to drive Process Deficiencies $H_{PD} = \sum_{i=1}^{Q} p_i \log_2 w_i$ to zero in (4.3) to reduce W to the minimum such that the amount of entropy in a factory is the minimum needed to produce any part number demanded by the market, *and no more*. In this context, the Toyota Production System minimizes entropy and hence waste for a given breadth of external product line

Summary: The market demand for variety imposes Entropy H_m upon the company through the revenue stream. The company responds to input H_m , transforming the m different products to Q different subsystems with corresponding entropy of component variety H_Q. The deficiencies of the internal processes add entropy H_{PD}. Lean Six Sigma initiatives drive $H_{PD} \to 0$ by driving $w_i \to 1$. Through Complexity reduction initiatives, H_Q is reduced limited only by the imagination of engineers while responding to profitable market demand or mission requirement in the case of government.

Section 7: Conclusion:

The empirically observed relationship (1.1) of waste to Work In Process, subject to the critique in Section 1, is explained as process entropy causing waste. We can now hypothesize a quantitatively defined the amount of cost reduction that will result from the addition of information though process improvement tools which reduce waste cost:

Hypothesis of the formula for Cost Reduction vs. Cycle time:

$$\textbf{\% Cost Reduction} \cong \mathbf{k_B}\log_{\mathbf{2}}(\alpha\tau_f/\tau_i) = \mathbf{k_B}\log_{\mathbf{2}}(\alpha W_f/W_i) \quad (7.1)$$

Where $\alpha = \frac{(\text{Revenue})_{\text{INITIAL}}}{(\text{Revenue})_{\text{FINAL}}}$ and

$k_B \cong 0.1$= tentative Boltzmann constant of Manufacturing

The Boltzmann constant k of thermodynamics converts entropy to energy/°K per unit of matter (the molecule). Likewise, the Boltzmann constant of Business k_B converts process entropy into percentage waste per unit of output. Formula (7.1) is a hypothesis based on empirical data which is consistent with consequences of Little's Law, and is subject to further tests to determine its validity as discussed below.

Section 8: Operational Procedure to Predict Cost reduction due to Process Improvement

1. Estimation of Non Value Add cost: A Value Stream Mapping process[3] will determine the value add and non value add cost of the process. Most of the non value add cost resides in labor and overhead cost, and may exceed 50% of those costs.

2. Measurement of Current Work In Process: The number of units of initial Work In Process W_i at each workstation or node and average completion rate must be determined. This information is typically available in manufacturing companies but must be obtained in non manufacturing processes. The minimum Work In Process will also be defined as $W_f = Q_f$ (number of internal part numbers needed to produce output after complexity reduction initiative) in manufacturing, and $W_f =$ A the number of steps in a transactional process. If the company is experiencing growth, the $\text{Revenue}_{\text{initial}}$ and $\text{Revenue}_{\text{final}}$ (after Y years of improvement) must be estimated to compute α in equation (7.1).

3. Defining the Maximum Cost Reduction Improvement goal: Given the data in 2 we can compute the maximum profit improvement that can result from process improvement

$$\text{Max \% Reduction in Cost of Goods Sold} = k_B \log_2\left(\frac{\alpha W_f}{W_i}\right)$$

This value, multiplied by current Cost of Goods Sold, should be less than the total non value add cost defined in 1 above as a check on the data.

4. Cost of Process Improvement: The cost of achieving W_f, or some practical alternative, should be the basis for a request for quotation to determine the invested

Capital \$C from the many process improvement consultants. The initiative should include Lean, Six Sigma, and internal and external Complexity reduction and. As an alternative, the company can consider the use of internal resources. Whatever W_f is chosen, the resulting % reduction should be multiplied current Cost of Goods sold to obtain the increase in profit ΔP.

5. Return On Capital: The company should estimate its share price multiple M of Earnings Before Interest, Taxes and Depreciation from the appropriate stock market (M=14 for the S&P 500 as of Oct 2007). Calculate the Return on Capital as:

$$\text{ROC \%} = \left(\frac{M(\Delta P)}{C}\right)^{\frac{1}{Y}} - 1$$

Where C was defined in 4. If the ROC exceeds 100% per year it will more than justify management execution of process improvement on a risk adjusted basis. As a benchmark, ROC % = 11% for the S&P 500 Oct 2007.

Observations:

Greatest Gains are from High Hanging, not Low Hanging Fruit: The Equation of Projected Cost Reduction (7.1) predicts that waste will follow a $\log_2(\alpha\tau_f/\tau_i)$ curve. Hence the gains from modest reductions of WIP are negligible but a reduction of WIP of greater than 70% will yield very significant returns. Experience shows that achieving such a goal requires top management engagement.

Increases in Complexity are to be avoided: While H_{PD} in (4.3) can always be reduced by process improvement, H_Q due to complexity cannot. As cycle time is reduced below 30% and the slope of the cost curve declines sharply (e.g. Case Study 2), any increase in cycle time and WIP due to increases in Q can result in a significant increase in cost. Discipline on Q is thus essential for cost dominance: as new part numbers or tasks are added, they should subsume or replace old part numbers or tasks, e.g., Toyota Motors and Scania Truck[11].

Section 9: Concluding Critique

Companies only report manufacturing WIP once per year in the inventory footnotes of their financial statements (SEC Form 10K in the US). The data is often polluted by individual factories sometimes defining WIP to include raw material. Moreover, reductions in WIP never exceed 50% at which point the equation on page 10 predicts large cost reductions.Companies do not report and generally do not capture the data on WIP or cost of transactional processes.

To obtain statistically significant data, the Institute of Business Entropy will, in cooperation with interested academics, instrument several companies such that:

1. WIP and unit volume can be sampled *monthly* during the improvement process

2. WIP can be reduced by>70% and consistently defined to exclude raw material and finished goods
3. Measure k_B in both manufacturing and transactional processes
4. Determine if the predicted logarithmic dependence of cost on cycle time is valid
5. Cost reduction is postulated as due principally to cycle time reduction, and growth in volume is normalized out using the constant α which is consistent with empirical data. A multiple regression will be compiled in properly instrumented companies to separate process improvement from volume effects in (7.1).

The subject matter of this paper as of the publication date is patent-pending. Pursuant to an agreement with Accenture Global Services, royalty free sublicenses may be granted for certain non-commercial purposes of research and testing by contacting mike@entropy2718.com

Academics who wish to participate in the study of these "live lab" companies and share data will be granted a royalty free sub-license to use patented information and freedom to publish their findings with adequate protection for participating companies. Interested parties who wish to submit a Grant Application for research funding or support of graduate students in relevant research should contact mike@entropy2718.com

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Online Appendix 1 Thermodynamic Review

Reference (2.3) We will begin with a brief study of the elimination of waste in an engine as this provides a heuristic for understanding waste elimination in a Microeconomic process.

Waste in an Engine: Carnot, followed by Clausius, reasoned that, in each cycle, an engine receives heat energy Q_H from a Hot combustion source at temperature T_H. With each power stroke of the piston, the engine transforms part of this input energy into useful work to drive a shaft. The rest of the input energy is expelled as waste energy Q_C to the environment at the Cold sink temperature of $T_C \approx 25°C$ at which point the cycle is complete and the engine is ready to receive more heat energy. Carnot discovered that a quantity known as the Entropy, $S=Q_H/T_H$ was drawn from the Hot source and at least that much Entropy was delivered to the Cold temperature sink:

$$\text{Entropy} = S = \frac{Q_H}{T_H} \leq \frac{Q_C}{T_C} \qquad \text{(A1.1)}$$

thus the minimum waste energy Q_C delivered to the Cold temperature sink is

$$\text{Waste} = Q_C \geq T_C S \qquad \text{(A1.2)}$$

In these equations, temperature is expressed in the absolute scale where 0°C=273°Kelvin. Minimum waste in an engine is proportional to entropy S that is output to the cold sink. The "greater than or equal to" sign is always "greater than" in any real engine due to the process being irreversible which creates additional waste. When a gas expands thru a nozzle virtually all the entropy created is irreversible. According to (A1.1) entropy falls as T_H increases. This discovery helped inform the development of engines, from the atmospheric engines of the 18th Century which operated at 3% efficiency and about 100°C to the modern gas turbines which operate at 40% efficiency and 3000°C. We posit that deriving the entropy flows of a Microeconomic process and the parameters related to entropy reduction will similarly inform the reduction of waste which is cost in a Microeconomic process.

The explicit expression for the entropy change of an ideal gas undergoing compression at a constant temperature in an engine is easily derived (e.g.,Fermi,E *Thermodynamics*)and will be useful in understanding the equivalent expression for process entropy (3.1):

$$\text{Change in Entropy} = \Delta S = \int \frac{dQ}{T}$$, but from the 1st Law of Thermodynamics

dQ=dU+pdV where Q=heat,T=Temperature,U=internal energy,P=Pressure,V=Volume

$$\Delta S = \int \frac{(dU+PdV)}{T} = \int \frac{(c_v dT+PdV)}{T} = \int \frac{PdV}{T}$$

c_v is the specific heat, n=number of moles

for isothermal processes where dT=0, and for an ideal gas PV=nRT, thus

$$\Delta S = \int \frac{nRTdV}{VT} = nR \int_{V_{Initial}}^{V_{Final}} \frac{dV}{V} = nR\log(V_{Final}/V_{Initial}) \quad \text{(A1.3)}$$

Similarly, if we compute the energy expended by an external force performing isothermal (constant temperature) compression on an ideal gas:

$$\text{Energy expended in compression} = \int_{V_i}^{V_f} P(-dV) = -\int_{V_i}^{V_f} \frac{nRT}{V} dV = -nRT\log(V_f/V_i) = -T\Delta S \quad \text{(A1.4)}$$

The energy of compression $-nRT\log(V_f/V_i)$ during the cold isotherm of a Carnot cycle represents the *minimum* amount of waste in an engine. Note the similarity of (A1.4) to (2.3) and (A2.9) in online Appendix 2

Online Appendix 2 Derivation: Effective Mass of a Process = W^2

Reference (2.3) To discover if entropy exists in Microeconomic processes, we will follow Clausius' derivation of (A1.4). We first transform Little's Law into a velocity equation by inversion:

$$\text{Process Velocity} = v = \frac{\text{Average Completion Rate}}{\text{No. of Units of Work In Process}} = \frac{1}{\tau} = \frac{D}{W} \text{ cycles/unit time} \quad \text{(A2.1)}$$

This velocity is the number of manufacturing cycles completed per unit time, or in the case of Product Development the number of design cycles/unit time. Clearly the velocity is inversely proportional to the Work In Process W and directly proportional to D. The first tenet of Toyota is that a Pull System[1] must be established such that not only the Average, but also the instantaneous completion Rate D = Market Demand. This having been accomplished, D is an exogenous variable driven by the market, and may be treated as constant to first order during periods comparable to the process cycle time.

As a first approximation, we will therefore assume that D is constant. We will then show in a subsequent paper that variable D does not affect the derivation of the equation of projected cost reduction.

The rate at which the velocity of a process in (A2.1) is accelerated is related to the rate at which W can be reduced, assuming that D is constant. Thus -dW/dt is a factor in the force shortening the process cycle time, i.e., accelerating the velocity of the remaining Work In Process. Taking the first derivative of (A2.1) we obtain:

$$\text{Process Acceleration} = a = \frac{dv}{dt} = -\frac{D}{W^2}\frac{dW}{dt} \text{ cycles/hour/hour} \quad \text{(A2.2)}$$

The Role of the factors in the Acceleration equation (A2.2):

Equation (A2.2) is the acceleration of the velocity with which the WIP completes a cycle of production. We will examine the role of the factors in equation (A2.2)

A. The $-\frac{dW}{dt}$ **factor** *in equation* (A2.2)*:*

We know that a reduction of WIP will accelerate the process, hence we relate this factor to an *external force* applied by process improvement to be discussed later which reduces WIP while maintaining D constant, hence accelerating velocity (A2.1).

B. The W^2 *term in* (A2.2)

Newton attributed the term "inertia" to mean "the innate force possessed by an object which resists changes in motion". The greater the inertial mass, the less will a body accelerate under a given external force such as $-\frac{dW}{dt}$. Looking at (A2.2) we conclude that, for a given magnitude of force $-\frac{dW}{dt}$, the larger W^2, the smaller the acceleration. In statistics we speak of the Probability Mass Function having the characteristics of Mass. In the same vein we tentatively associate W^2 as having the characteristics of the inertial mass of the process. One might intuitively expect the inertial mass of a process to be directly proportional to W. However, each unit of WIP can only advance *on average* if all those ahead of it also advance, as well as all those behind it. Thus each unit of WIP is, *on average*, coupled to all the other units of WIP through Little's Law. This coupling is analogous to an inductor, in which each turn is coupled to all the other turns in the inductor, leading to self inductance proportional to the *square* of the number of turns rather than directly with the number of turns. Since WIP W is a dimensionless number, so is the inertial mass of a process, W^2. However, unlike the dynamics of particles, the acceleration of WIP is determined, not by its mass in kilograms, but by the total number of units of WIP in the process. Since the inertial mass of a mechanical body is measured in kilograms, several readers have suggested that the word "mass" cannot be used in its technical sense to describe a process. Others have suggested no distinction is necessary since Probability *Mass* function is not measured in grams, nor is the *Power* Spectral Density measured in watts. We propose to make the distinction between mechanical inertial mass and process inertial mass by coining the term *Prinertia* for Process inertia.

$$\text{Process Inertia} = \text{Prinertia} = W^2 = M_M = \text{Mass}_{\text{Microeconomic}} \qquad \text{(A2.3)}$$

Where we denote a Microeconomic analogy by the subscript M

Consistency of $M_M = W^2$ with Newton's 2nd Law: We have tentatively assumed that W^2 is the Prinertia of a process, and will determine if the derivation thus far is consistent with Newton's Second Law. From (A2.1) we have

$v = \frac{D}{W}$, and Momentum = p = Mv = $W^2\left(\frac{D}{W}\right)$ = DW

Using the Variational Principle known as the Principle of Least Action, we consider p and v to be independent variables in phase space that take on the values $p_i v_i$ at t_i and $p_f v_f$ at t_f, we have:

$$\text{Action}=\int_{t_i}^{t_f} \text{pvdt} = \int_{t_i}^{t_f} DW\left(\frac{D}{W}\right)dt = D^2\left(t_f - t_i\right) \text{ all of which are constant, hence:} \qquad (A2.4)$$

$$\Delta\left(\text{Action}\right) = 0 = \Delta\int_{t_i}^{t_f} \text{pvdt} = \int_{t_i}^{t_f} \Delta\left(D^2\right)dt = \int_{t_i}^{t_f} \left(0\right)dt = 0 \text{ since D is an exogenous constant}$$

Since the variation in Action is zero, the Euler-Lagrange criterion is satisfied, and Newton's Laws are the equations of motion of a process

C. The D factor in (A2.2)*:*

While the role of W^2 and $-dW/dt$ in process acceleration is clear, the role of the D factor, unit demand per unit time, is not obvious. There are two possible parsings of(A2.2). To determine whether the D factor in (A2.2) is part of force $-dW/dt$ or Prinertia W^2, we will calculate the "energy" to accelerate the WIP from the initial velocity to a faster velocity. We will require that the resulting units of measure of "energy" expended by the external force be in appropriate units of $\frac{1}{2}(Mv^2)$. The parsing of (A2.2) which fails to achieve this criterion will be rejected. This will thus determine if the D factor is part of Force or Prinertia. Given that $M = W^2$ is dimensionless in a process, process energy

$$\frac{1}{2}\left(Mv^2\right) = \frac{1}{2}\left(W^2\left(\frac{D}{W}\right)^2\right) = \frac{1}{2}\left(D^2\right) \qquad (A2.5)$$

will be measured in terms of a velocity squared which is $(\text{units/unit time})^2$.

When we speak of the kinetic energy of moving Work In Process, we cannot use the term "energy" in its strict technical sense since Energy is measured in Joules. Moreover, the Joules expended on WIP, and likewise the dollar value of WIP, has nothing to do with the velocity of WIP which is solely governed by Little's Law (2.1). We will therefore employ the coined term Process energy or "Prenergy" to describe the process equivalency to $\frac{1}{2}(Mv^2)$ which results from the external force of process improvement. Let us follow a unit of WIP down the process. Process improvement is continually reducing setup time, batch size and hence WIP W or its equivalent in transactional processes as discussed in ref 18. In time dt, the unit of WIP will, on average, be slightly accelerated as it moves a distance ds down the process, reducing τ, hence increasing the number of production cycles per unit time.

Test of the Parsing of* (A2.2) *as $\left(\text{Mass}\right)_M = W^2$, and $\left(\text{Force}\right)_M = -D\frac{dW}{dt}$

The amount of "Prenergy" applied by the external force of improvement in accelerating the WIP is:

$$\Delta \text{Prenergy} = \int_{S_i}^{S_f} F ds\ , \qquad \text{(A2.6)}$$

but if v is the velocity of the WIP, then ds=vdt,
and with D a factor in Force we have:

$M_M \rightarrow$ Prinertia $=W^2$ which is dimensionless since W is a dimensionless number

$$\text{and } F = -D\frac{dW}{dt} \qquad \text{(A2.7)}$$

then with $v = \frac{D}{W}$, $ds = \frac{D}{W}dt$ where ds is movement down the process from one workstation to another, therefore:

$$\Delta \text{Prenergy} = \int_{S_i}^{S_f} F ds = \int \left(-D\frac{dW}{dt}\right)\left(\frac{D}{W}dt\right) = -D^2 \int_{W_i}^{W_f} \frac{dW}{W} = -D^2\left(\log W_f - \log W_i\right) \qquad \text{(A2.8)}$$

Which has the correct units of measure per (A2.5). In a series of four additional independent tests in the subsequent paper including:

1. Maxwell's velocity and the connection of Process Entropy to Information.
2. Entropy of a mixture of Ideal Gases consistent with Process Entropy.
3. Economic Process Improvement consistent with Thermodynamics of the Nozzle
4. Equations of Process flow consistent with Ideal Gas flow

we show that no other parsing of (A2.2) between mass and force yields the correct units of measure or is consistent with mechanics and thermodynamics. The right side of (A2.8) resembles the energy expended by an external force in the compression of an ideal gas at constant temperature per (A1.4) with D^2 tentatively taking the place of temperature. The second factor on the right side of equation (A2.8) is tentatively the entropy change of an economic process. Using Little's Law (2.1), (A2.8) can also be written in the form of (2.3) as

$$\Delta \text{Prenergy} = -D^2 \log\left(\frac{W_f}{W_i}\right) = -D^2 \log\left(\frac{D\tau_f}{D\tau_i}\right) = -D^2 \log\left(\frac{\tau_f}{\tau_i}\right) \qquad \text{(A2.9)}$$

Online Appendix 3 Derivation: Entropy of Work in Process

Reference (3.1) We will first derive an expression for log W with Q = 2 and then generalize:

$W = w_1 + w_2$

Thus far we have used the natural logarithm log W= $\log_e W$. We will take the $\log_2 W = 1.44 \log_e W$ and use the conversion factor where needed.

This allows results to be stated in bits rather than nats. We can write:

$$\log_2 W = \frac{w_1+w_2}{W}\log_2 W = \frac{w_1}{W}\log_2 W + \frac{w_2}{W}\log_2 W = -\frac{w_1}{W}\log_2\left(\frac{1}{W}\right) - \frac{w_2}{W}\log_2\left(\frac{1}{W}\right),$$

we will now add 0 + 0

$$\log_2 W = -\frac{w_1}{W}\log_2\left(\frac{1}{W}\right) - \frac{w_2}{W}\log_2\left(\frac{1}{W}\right) + \left(\frac{w_1}{W}\log_2 w_1 - \frac{w_1}{W}\log_2 w_1\right) + \left(\frac{w_2}{W}\log_2 w_2 - \frac{w_2}{W}\log_2 w_2\right)$$

$$\log_2 W = -\frac{w_1}{W}\log_2\left(\frac{w_1}{W}\right) - \frac{w_2}{W}\log_2\left(\frac{w_2}{W}\right) + \frac{w_1}{W}\log_2 w_1 + \frac{w_2}{W}\log_2 w_2$$

which can be generalized from Q = 2 to Q

different types which comprise W by defining the Probability that a unit of WIP is the i^{th} product as $p_i = \frac{w_i}{W}$

$$\text{Process Entropy} = \log_2 W = -\sum_{i=1}^{Q} p_i \log_2 p_i + \sum_{i=1}^{Q} p_i \log_2 w_i = H_Q + \sum_{i=1}^{Q} p_i \log_2 w_i \qquad \text{(A3.1)}$$

$\log_2 W = H_Q + \varepsilon \log_2 w_i$, where ε is the expectation:

$$\varepsilon \log_2 w_i = \sum_{i=1}^{Q} p_i \log_2 w_i$$

$$W = 2^{H_Q + \varepsilon \log_2 w_i} \qquad \text{(A3.2)}$$

Online Appendix 4 Information Theory Review

Reference Section 4.We will show that these initiatives inject Information into the process, and that Information is in fact Negative Entropy which reduces waste. Let's first define what we mean by Information. Information tells us something unexpected, i.e., there is a "surprise". The Ford Model T line held no surprise…every car coming off the line was an identical Black Model T, every flywheel magneto was the same with 100% probability, and hence no information was to be gained by looking at the next car or component coming off the line. But what if you were told that it was July 4th in Dallas and there was four feet of snow on the ground…this highly improbable event would be very surprising and hence convey huge information. Therefore we conclude that the *amount* of Information is inversely related to the probability of the event. It is also reasonable that, whatever the functional form of Information may be , if two independent events, 1 and 2 happen, the total information is the sum of their separate Information I_1 and I_2, i.e., $I_{1\&2}=I_1+I_2$. But the probability of independent events 1 and 2 both happening is the product of their probabilities $p_{1\&2}= p_1p_2$. So we need some function for Information I such that: $I_{1\&2}(p_1p_2) = I_1(p_1)+I_2(p_2)$ and the only function which satisfies this requirement is $I = \log(p)$ since $\log(p_1p_2) = \log(p_1)+\log(p_2)$. Therefore $I(p)=\log(p)$. But since we want the Information to be larger if the probability is smaller we will define $I(p)=\log(1/p) = -\log(p)$ which still satisfies $\log(1/p_1p_2)=\log(1/p_1)+\log(1/p_2)$. The *average* amount of information among N choices is, like any other average, just the sum of the probability of each choice times the value of each choice:

$$H= -\sum_{i=1}^{N} p_i I_i = -\sum_{i=1}^{N} p_i \log_2 p_i \qquad (A4.1)$$

Equation (A4.1) is known as the Shannon equation of Information. Note that it is identical to the entropy of statistical mechanics per ref 12

$$S= -k\sum_{i=1}^{N} p_i \log_2 p_i$$

but with k=1. We show in Appendix 4 that the thermodynamics of process velocity yields k=1 automatically. Hence Microeconomic processes are accelerated by the addition of information. We will now show how the market place transmits information to the company. Assume a company produces two products, product 1 in quantities d_1 per WIP inventory turn, and product 2 in quantities d_2 per WIP inventory turn, where $d_1 + d_2 = D$ total units produced per WIP inventory turn. The actual demand of the market for the two products is random, and results in a variety of possible sequences such as:

1121221122212212
2211212211121221
2122122111211212
, etc.

The market makes D Choices per WIP inventory turn of either 1 or 2. Each sequence is a "state" of the market or complexion in the sense of Gibbs .The number of *distinct* states or "messages" sent by the market, to be satisfied by the company, is calculated by the usual combinatorial formula[1]:

$$\text{Number of Distinct Messages per inventory turn}=M=\frac{D!}{d_1!d_2!}=\binom{D}{d_1}=\frac{D!}{d_1!\left(D-d_1\right)!} \qquad (A4.2)$$

We will follow Boltzmann by taking the logarithm of the number of states, which in the Microeconomic case is the number of distinct messages from the market:
According to Stirling's formula, to first order[1]:

$$\log_2 D! \cong D\log_2 D-D, \text{ note that } D=(D-d_1)+d_1=d_2+d_1$$

$$\log_2 M=\left(D\log_2 D - d_1\log_2 d_1-\left(D-d_1\right)\log_2\left(D-d_1\right)\right)$$

$$\log_2 M=\left(\left((D-d_1)+d_1\right)\log_2 D - d_1\log_2 d_1-\left(D-d_1\right)\log_2\left(D-d_1\right)\right)$$

$$\log_2 M=-\left((D-d_1)\log_2\left(\frac{D-d_1}{D}\right)+n_1\log_2\left(\frac{d_1}{D}\right)\right),\text{multiplying by }\frac{D}{D},\text{obtain:}$$

$$\log_2 M=-D\left(\left(\frac{D-d_1}{D}\right)\log_2\left(\frac{D-d_1}{D}\right)+\left(\frac{d_1}{D}\right)\log_2\left(\frac{d_1}{D}\right)\right),\text{let } p_1=\frac{d_1}{D},p_2=\frac{D-d_1}{D}$$

$$\log_2 M = D\left(-\left\{p_1\log_2 p_1 + p_2\log_2 p_2\right\}\right) \rightarrow D\left\{-\sum_{i=1}^{m} p_i\log_2 p_i\right\} = DH_m \text{ for m products,}$$

$$M = 2^{DH_m} = \text{Number of Distinct Messages M per WIP inventory turn} \qquad (A4.3)$$

$$DH_m = \log_2 M$$

$$D = \frac{\log_2 M}{H_m} \qquad (A4.4)$$

Notice that Shannon's equation for Information emerged naturally. The market is making D variety choices per WIP inventory turn, selected from one of the m products, each of the D events per turn containing information of H_m bits. The M messages corresponds to the number of unique states per inventory turn..

$$H_m = -\sum_{i=1}^{m} p_i \log p_i = \text{Shannon Information in Bits per Choice}$$

$$\text{Transmission Rate of Market } \rightarrow DH_m \rightarrow \left(\frac{\text{Choices}}{\text{WIP Turn}}\right)\left(\frac{\text{Bits}}{\text{Choice}}\right) \rightarrow \text{ Bits per WIP Turn} \qquad (A4.5)$$

Online Appendix 5 Process Improvement Review

How Lean Six Sigma and Complexity Reduction add information to a process

In Section 7, we asserted that a Microeconomic process is improved by the addition of information. We will illustrate this point with a specific example which relates process improvement to information. When processing in batches of quantity B, how much information is added by selecting a given product to setup and run? Let us assume that a factory consists of A workstations, each of which processes on average Q/A = N part numbers. Clearly if there are N products produced at a given workstation, the decision to select one creates H_N bits of information. However, the probability is 1 of running that product for the rest of B-1 units in the batch. Therefore, the B-1 units add zero information. As the setup time is cut in half, the batch size can be cut in half and still maintain the same production rate according to (5.1) .Now however we add information twice as often because we select the particular product of the N possibilities twice as often. In general, the information supplied to the process is thus:

$$I_N = \text{Information in production of N Products per month} = \frac{N}{B} H_N$$

$$B \geq \frac{sD}{1 - X - \zeta D} + 1 \text{ according to Patell-George, ref 19}$$

where s=Setup Time, X=scrap rate, ζ=Processing time/Unit, D=total demand in units/unit time (B will increase as a function of variation of parameters via simulations) hence

$$I_N = \frac{N}{\left(\frac{sD}{1-X-\zeta D}+1\right)} H_N \rightarrow NH_N \text{ as } s \rightarrow 0 \qquad \text{(A5.1)}$$

and for A workstations, AN=Q which is necessary to produce m external products for customers

$$ANH_N \rightarrow QH_N \rightarrow H_m \qquad \text{(A5.2)}$$

Thus the goal of the Toyota Production system to respond "Just In Time" and produce only what is needed when it is needed is equivalent to an information flow within the factory which matches market demand. In regard to entropy due to average WIP, Lean Six Sigma process improvement results in: :

$$S_{initial} - S_{final} = c_0 k w \varepsilon_Q \left(\log_2 w_{j initial} - \log_2 w_{j final}\right) \qquad \text{(A5.3)}$$

$$S_{initial} - S_{final} = c_0 k w \varepsilon_Q \left(\log_2\left(\frac{s_{initial} D}{1-X_{iniital} - \zeta_{initial} D}+1\right) - \log_2\left(\frac{s_{final} D}{1-X_{final} - \zeta_{final} D}+1\right)\right) \qquad \text{(A5.4)}$$

Applying *Lean* initiatives such as driving s→0 drives entropy related to WIP→0 and leaves only the entropy related to H_m due to the *Complexity* of parts. Applying Six Sigma to drive X→0 is often of equal power. The addition of information by Lean Six Sigma as a means of reducing entropy is merely one example of a general theory propounded by the Physicist Leon Brillouin in which he coined the term *Negentropy* for Information since it is Negative Entropy as is seen in (A5.4) as the amount of entropy subtracted by addition of process information. Although the specific process improvement tools change, the same conclusion applies to transactional processes. For a manufacturing process the minimum WIP is clearly Q, that is, one of every internal item necessary to produce the current mix of products. For a transactional process the minimum WIP is one task per step in the process, or A since there are A steps. The minimum WIP may never be attainable from a practical viewpoint, but has provided the purposive thrust for Toyota among others.

[1] Director, Institute of Business Entropy, Dallas, TX http://www.entropy2718.com/; Founder, The George Group. Bio at http://www.georgegroup.com/michael_george.php click on “Continue to George Group”

[2] George, M, Montgomery, D , and Gooch, J *America Can Compete* p18, (1987)

[3] See for example: George, Michael L et al: *Lean Six Sigma*(2002), *Lean Six Sigma for Service*(2003),*Conquering Complexity*(2004), *Fast Innovation*(2005),*Lean Six Sigma Pocket Tool Book*(2005) all by McGraw-Hill. The extension of the original PK equation includes the factor K which is an approximation in Mike Harrison’s Stanford Queueing Theory T363 notes. The inclusion of X assumes rework time=original processing time.

[4] Hopp,W and Spearman M *Factory Physics*, Irwin(1996)

[5] Work In Process are the units of work that have been released into production from Raw Material but have not yet been delivered to a customer and hence includes Finished Goods, and appears in a footnote in the 10K of public corporations in manufacturing companies. In non manufacturing and government processes, the number of tasks in process at each step in the process must be empirically determined

[6] Feynmann, R. P., *Statistical Mechanics*, p. 6, equation (1.3) Addison-Wesley (1998)

[7] George, M and Wilson,S *Conquering Complexity in Your Business* pp 14-17,McGraw-Hill(2004)

[8] US Patent 6,993,492 issued Jan 31, 2006

[9] Taylor, F *Principles of Scientific Management*, NuVision, (2007 reprint)

[10] MIT LAI paper 2007 op cit

[11] H. Thomas Johnson, and Anders Brom , *Profit Beyond Measure*, Free Press, (2000)