Determination of Force on and Mass of a Process using the Principle of Least Action

Newton's 2nd Law requires $M = W^2$: From Little's Law we know that the force that accelerates process velocity is proportional to the rate of WIP reduction, $-dW/dt$. From Equation (8), the constant of proportionality on the force and the effective Mass of the process are not obvious. We therefore need two independent conditions to make this determination. We know that the velocity of the process is $v = G/W$, and that momentum is $p = Mv = M(G/W)$.

Condition 1: Principle of Least Action[14]. To obey Newton's 2nd Law, the force accelerating the process must satisfy:

$$\Delta(\text{Action}) = \Delta\left(\int_{ti}^{tf} pv\,dt\right) = \Delta\left(\int_{ti}^{tf} Mv^2 dt\right) = \Delta\left(\int_{ti}^{tf} M\left(G^2/W^2\right)dt\right) = 0$$

Now, since $\Delta W \neq 0$ due to process improvement, for $\Delta(\text{Action}) = 0$, we must have a functional form for the mass such that it will cancel out the $1/W^2$ in the Action integral, leaving only constants. Thus, either $M = W^2/G$ and G is a factor in Mass, or $M = W^2$ and G is a factor in the force. In either case, $M = kW^2$:

$$\Delta(\text{Action}) = \Delta\left(kG^2 \int_{ti}^{tf} dt\right) = \Delta\left(kG^2\left(t_f - t_i\right)\right) = 0$$

Since the variation in Action is zero, the Euler-Lagrange criterion is satisfied, $M = kW^2$ is proved, allowing the integration of (11).

Condition 2: Units of Measure. Although not required for the integration of (11), we wish to determine if $M = W^2$ or $M = W^2/G$ by applying the definition of momentum to both cases and examining the resulting units of measure:

$$M = W^2:\ p = Mv = W^2\left(G/W\right) = GW \rightarrow \text{unit}^2/\text{unit time}$$

$$\text{or } M = W^2/G:\ p = Mv = \left(W^2/G\right)\left(G/W\right) = W \rightarrow \text{units}$$

Since momentum has a unit of measure per unit time, $M = W^2$ leads to a unit of measure appropriate to momentum Mv, whereas W^2/G does not. Note that this derivation of Mass $= W^2$ is independent of the derivation (8) that began with Little's Law (6). Substituting $M = W^2$ into (8) yields (9).